Collective and chaotic motion in self-bound many-body systems

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We investigate the interplay of collective and chaotic motion in a classical self-bound N-body system with two-body interactions. This system displays a hierarchy of three well separated time scales that govern the onset of chaos, damping of collective motion and equilibration. Comparison with a mean-field problem shows that damping is mainly due to dephasing. The Lyapunov exponent, damping and equilibration rates depend mildly on the system size N.

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Self-bound many-body systems like the atomic nucleus display regular, collective phenomena as well as chaotic behavior [1,2]. The giant dipole resonance, for example, constitutes a *regular* oscillation of the protons against the neutrons albeit it is excited at energies where the spectrum displays fluctuations which are typical for *chaotic* systems [3]. This interesting interplay of collective and chaotic motion and the effects of chaotic dynamics on the damping and dissipation of nuclear excitations is a matter of intense research [4–14] but still not fully understood.

Several authors [4,5,9,10,13] have addressed equilibration and damping of collective motion by coupling a slowly moving collective degree of freedom (typically the wall of a container) to fast moving independent particle degrees of freedom. Within these models, collective degrees of freedom are neither constructed from single-particle coordinates nor the result of a self-consistent mean field. In the case of driven walls this may lead to energy conservation problems. Bauer *et al.* [14] treated a more realistic model where the collective motion was coupled self-consistently to the single-particle motion and found the coexistence of undamped collective motion and chaotic single-particle dynamics. This observation is interesting with view on low-lying collective excitations but cannot apply to the case of the giant dipole resonance. Furthermore, the classical models discussed in the literature suffer from the absence of a two-body force and rotational symmetry. In this work we are particularly interested in the many-body aspects of the problem and the role of the two-body interaction opposed to the mean-field. To this purpose we study a classical self-bound many-body system with the following characteristics: two-body surface interaction; chaotic dynamics; collective mode constructed from single-particle degrees of freedom.

Let us consider the model Hamiltonian

$$H = \sum_{i=1}^{N} \frac{\vec{p}_i^{\,2}}{2m} + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|), \tag{1}$$

where $\vec{r}_i$ is a two–dimensional position vector of the i-th nucleon and $\vec{p}_i$ is its conjugate momentum. The interaction is given by

$$V(r) = \begin{cases} 0 & \text{for } r < a, \\ \infty & \text{for } r \geq a. \end{cases} \tag{2}$$

The particles thus move freely within a convex billiard in $2N$-dimensional configuration space and undergo specular reflection at the wall. This corresponds to the basic picture of nucleon motion in a nucleus: they move in a common flat-bottom potential, and interact mainly at the surface. The Hamiltonian (1,2) achieves these features while retaining the two-body interaction. Total momentum $\vec{P}$, energy E and angular momentum L are conserved. Classical phase space structure is independent of energy since a scaling $\vec{p}_i \to \alpha\vec{p}_i$ simply leads to a rescaling of $E \to \alpha^2 E, \vec{P} \to \alpha\vec{P}$, and $L \to \alpha L$. We set $m = a = \hbar = 1$. Thus, energies, momenta and times are measured in units of $\hbar^2/ma^2$, $\hbar/a$ and $ma^2/\hbar$, respectively [1]. In what follows we fix the total momentum $\vec{P} = 0$, the angular momentum $L = 0$ and set the energy $E = N$.

The $N = 2$ system is integrable and chaotic dynamics may exist for $N \geq 3$. To study the classical dynamics of the N-body system we compute the Lyapunov exponent using the tangent map [15]. The time evolution of the N-body system is done efficiently by organizing collision times in a partially ordered tree and requires an effort $\mathcal{O}(N \ln N)$ [16]. We draw several initial conditions for fixed N at random from the $(E = N, \vec{P} = 0, L = 0)$-shell and follow their time evolution for several hundred Lyapunov times. This ensures good convergence of the Lyapunov exponents. The results of several runs are listed in Table I (mean values and RMS deviations). For $N = 3$ we traced 7×10^4 trajectories, all of them being unstable with respect to small initial deviations. Thus, we expect more than 99% of phase space to be chaotic. For larger N we traced fewer trajectories and have less statistics. However, *all* followed trajectories have positive Lyapunov exponents. This suggests that the dynamics of the N-body system is dominantly chaotic. We note that the N-body system possesses marginally stable orbits corresponding to configurations where $N - 2$ particles are at

[1] We introduced $\hbar$ to set an energy scale.

rest. However, these configurations are of zero measure in phase space. The reliability of the numerical evolution was checked by comparing forward with backward evolution. Moreover, total energy, total momentum and angular momentum were conserved to high accuracy. As a further check we computed the Lyapunov exponents using an alternative method [17] and found good agreement with the results displayed in Table I.

We next consider the time evolution of collective motion. To this purpose we define a set of initial conditions (i.e. phase space points) of the N-body system that lead to collective motion corresponding to the giant dipole resonance. In passing we mention that one could also use collective coordinates introduced by Zickendraht [18] or consider motion close to invariant manifolds of the rotationally symmetric many-body system [19]. Let us draw the momenta $\vec{p}_i$ at random from uniform probability distributions that vanish outside of the domains $(p_x, p_y) \in [-\Delta q_x, \Delta q_x] \times [\pm q - \Delta q_y, \pm q + \Delta q_y]$, with $+$ or $-$ sign for particles $i = 1, \ldots, N/2$ (e.g. protons) and particles $i = N/2 + 1, \ldots, N$ (e.g. neutrons), respectively. We rescale the energy to $E = N$. The initial positions $\vec{r}_i, i = 1, \ldots, N-1$ are drawn at random to lie inside a circle of radius $a/2$; the position of the Nth particle is chosen such that the total angular momentum vanishes; the center of mass is subtracted. For $\Delta q_x \approx \Delta q_y \ll q \approx \sqrt{2mE/N}$ one obtains initial conditions that correspond to the motion of all protons against all neutrons and therefore may be identified with the dipole giant resonance[2]. In what follows we use $\Delta q_x = \Delta q_y \approx q/10$.

We are interested the time evolution of the dipole-moment

$$\vec{D} = \frac{2}{N} \sum_{i=1}^{N/2} (\vec{r}_i - \vec{r}_{N-i}). \quad (3)$$

Fig. 1 shows the time dependence of the dipole-moment for a system of size $N = 220$ and one collective initial condition. The longitudinal component D_y exhibits a few damped oscillations that eventually turn into erratic fluctuations of small amplitude. On the same time scale the transverse component D_x increases in amplitude from approximately zero and turns into similar erratic fluctuations. These fluctuations decrease with increasing system size N, indicating that they are of statistical nature. The first two periods of the time evolution can be fitted to an exponentially damped harmonic oscillation yielding the period τ and the damping rate γ. The results of several runs (mean values and RMS deviations are listed in Table II.

[2]Our choice of zero angular momentum is justified in the framework of classical mechanics.

For the equilibration we study the distribution of single-particle energies $\epsilon_j, j = 1 \ldots N$ during the time evolution of the dipole mode. The moments $I_\nu \equiv N^{-1} \sum_{j=1}^{N} \epsilon_j^\nu$ of this distribution may be compared with the corresponding equilibrium values obtained from a Boltzmann distribution $\rho^{(B)}(\epsilon) = 1/[kT \exp(\epsilon/kT)]$ at a temperature $kT = E/N = 1$, i.e. $I_\nu^{(B)} = \nu!(kT)^\nu$. Fig. 2 shows the first nontrivial moment I_2 averaged over many runs to reduce statistical fluctuations. This moment approaches its equilibrium value exponentially fast at a rate α (listed in Table II). The saturation at long times seems to be due to the finite number of particles. Once the system has equilibrated, the single-particle momenta are Maxwell-Boltzmann distributed (See Fig. 3). We also computed the Lyapunov exponent λ of the many-body trajectory corresponding to the collective dipole motion and listed the results (mean values and RMS deviations) of several runs in Table II. The Lyapunov exponents agree well with the previously computed ones listed in Table I. This shows that the dynamics of the many-body system is also chaotic in the region of phase space that corresponds to the collective motion. Table II displays a hierarchy of well separated rates $\lambda > \gamma > \alpha$ with $\lambda \gg \alpha$. This hierarchy is particularly pronounced for large N since λ (α) increases (decreases) with increasing N. The damping and equilibration rates differ for the following reason. While the dipole mode is damped out once the single-particle momentum distribution exhibits spherical symmetry, equilibration requires also a considerable change in the radial momentum distribution. Note that the dipole moment $\vec{D}$ and the moments I_ν of the single-particle energy distribution effectively test the angular and radial momentum distribution, respectively. Our results show that a generic self-bound many-body system with two-body interactions displays a fast equilibration in the angular components opposed to a slow equilibration in the radial components of the single-particle momenta. This observation is also of interest with view on heavy ion collisions.

To examine the role of the two-body interaction more closely, we consider a system of N independent particles moving inside a circular billiard of diameter a. This system is integrable since single-particle energies and angular momenta are conserved quantities. It can be taken as a mean-field approximation of our Hamiltonian [20] and is motivated by the observation that the surface of the interacting many body-system (i.e. the points in two-dimensional configuration space where interactions occur) becomes sharper with increasing numbers of particles and may roughly be approximated by a circle of diameter a. As before, we take initial conditions at random from the region that corresponds to collective motion and compute the dipole-moment (3) as a function of time. Fig. 4 compares the evolution of the longitudinal dipole-moment D_y obtained for the mean-field model

with the result obtained for the interacting many-body system. One observes damped oscillations with a period and a damping rate that are slightly larger than for the interacting system. The mean-field Hamiltonian therefore captures the damping of collective motion quite accurately. In the integrable system, damping is due to the dephasing of single-particle oscillations that is induced by the collisions with the surface, i.e. the directions of single-particle momenta get randomized on a time scale $1/\gamma$. The vadility of the mean-field picture is also confirmed by the observation that the periods and damping rates display no significant N-dependence. This is, however, different for the Lyapunov exponents and equilibration rates. These do depend on N through the presence of the two-body interaction. Note that our observation of N-independent frequencies $\omega = 2\pi/\tau$ and damping rates γ leads to a simple scaling law. Keeping the average single-particle energies E/N fixed and scaling the diameter of the N-body system as $a \propto N^{1/3}$ yields $\omega, \gamma \propto N^{-1/3}$. These results are in semi-quantitative agreement with experimental data [6], i.e. $\hbar\omega \propto N^{-1/3}$ in heavy nuclei, and $\gamma \propto N^{-[1/3\ldots 2/3]}$.

We also considered initial conditions of the interacting many-body system with larger momentum spread $\Delta q_x \approx \Delta q_y \approx q$, i.e. the initial momentum distribution is closer to an equilibrium distribution. This situation is closer to the perturbative excitation of the nuclear giant resonance. As expected, one finds a collective oscillation of smaller amplitude and comparable period, showing that the observed phenomena are robust. The determination of the damping rate is, however, difficult since amplitide and statistical fluctuations are roughly of the same magnitude.

In summary, we have considered damping and equilibration of collective motion in a self-bound N-body systems with two-body (surface) interactions. This system is predominantly chaotic and exhibits damped collective motion that leads to equilibration. There is a hierarchy of three well separated time scales starting with the onset of classical chaos at short times, damping of the dipole mode at intermediate times and equilibration at long times, respectively. The damping is mainly due to dephasing and may be understood quite accurately in a mean-field picture of non-interacting particles. Consequently, periods and damping rates show no significant N-dependence. Equilibration, however, requires the randomization of the magnitudes of single-particle momenta and a two-body interaction. Lyapunov exponents and equilibration rates depend mildly on N. The presented model exhibits a rather homogeneous phase space structure and the main characteristics of self-bound many-body systems like atomic nuclei. Our results show that it captures important features of collective motion in such systems.

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N	3	100	150	220	330	500
λ	0.963	1.506	1.561	1.62	1.67	1.75
$\Delta\lambda$	0.004	0.017	0.020	0.02	0.02	0.02
Runs	70000	100	100	50	30	10

TABLE I. Lyapunov exponents (mean values λ and RMS deviations $\Delta\lambda$) for N-body systems with unit single-particle energy $E/N = 1$. Results are obtained from the listed number of runs, λ is given in units of $\hbar/ma^2$.

N	100	150	220	330	500
λ	1.52 $\pm$0.02	1.57 $\pm$0.01	1.61 $\pm$0.03	1.67 $\pm$0.02	1.73 $\pm$0.02
γ	0.39 $\pm$0.05	0.38 $\pm$0.08	0.42 $\pm$0.06	0.39 $\pm$0.03	0.37 $\pm$0.04
α	0.21	0.19	0.15	0.13	0.10
τ	1.77 $\pm$0.05	1.77 $\pm$0.04	1.76 $\pm$0.03	1.77 $\pm$0.02	1.77 $\pm$0.01

TABLE II. Damping rates γ, periods τ, equilibration rates α and Lyapunov exponents λ of the collective mode. (γ,τ,λ: Mean values and RMS deviations are obtained from ten runs for each listed N-body system. α: exponent obtained from average of many runs.) The single-particle energy is $E/N = 1$; $\tau, 1/\lambda, 1/\gamma$ and $1/\alpha$ are given in units of $ma^2/\hbar$.

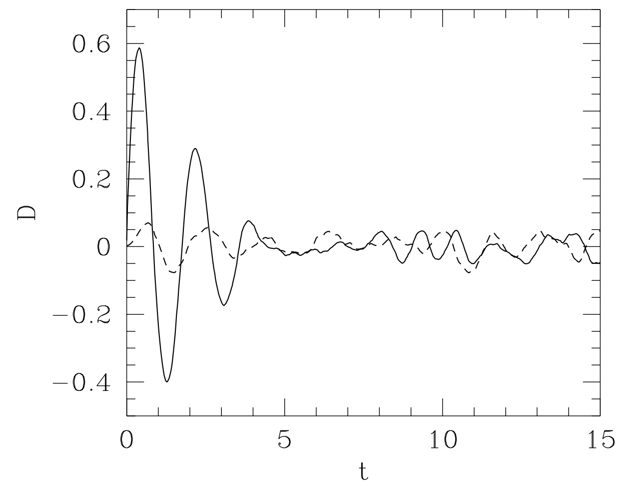

FIG. 1. Time evolution of the dipole-moment (3) for a system of $N = 220$ nucleons at energy $E = N$. (Full line: longitudinal D_y-component; dashed line: transversal D_x-component.) Time and dipole-moment are given in units of $ma^2/\hbar$ and a, respectively.

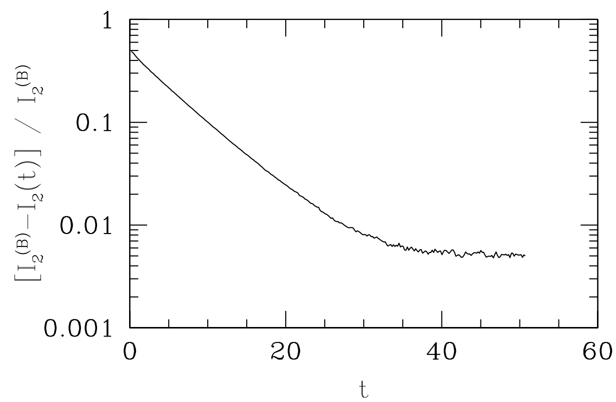

FIG. 2. Second moment $I_2(t)$ of the single-particle energy compared to its equilibrium value $I_2^{(B)}$ for $N = 220$. Time in units of $ma^2/\hbar$.

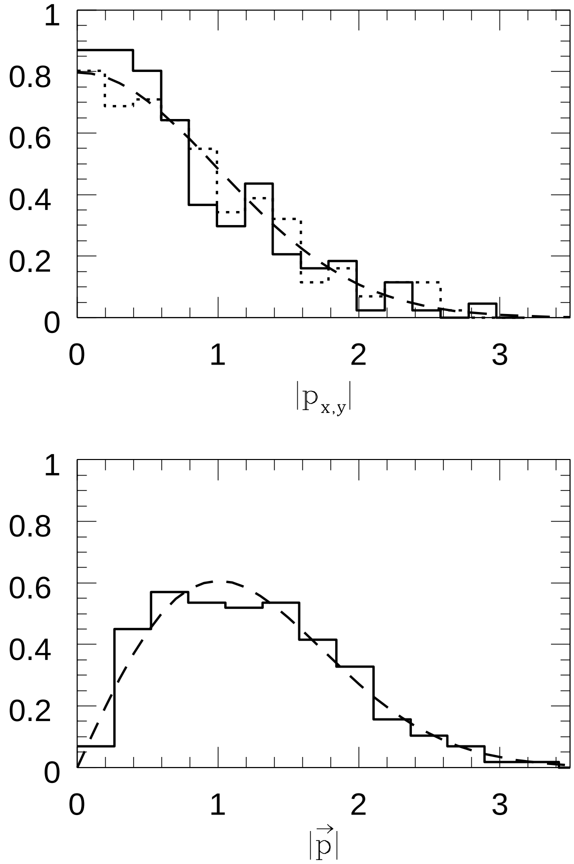

FIG. 3. Top: Normalized momentum distributions of single particle momenta $|p_x|$ (top: full line), $|p_y|$ (top: dotted line) and $|\vec{p}|$ (bottom: full line) after the equilibration of the collective motion compared to Maxwell-Boltzmann distributions with temperature $kT = E/N = 1$ (dashed lines). A system of $N = 220$ nucleons is considered; momenta are given in units of $\hbar/a$.

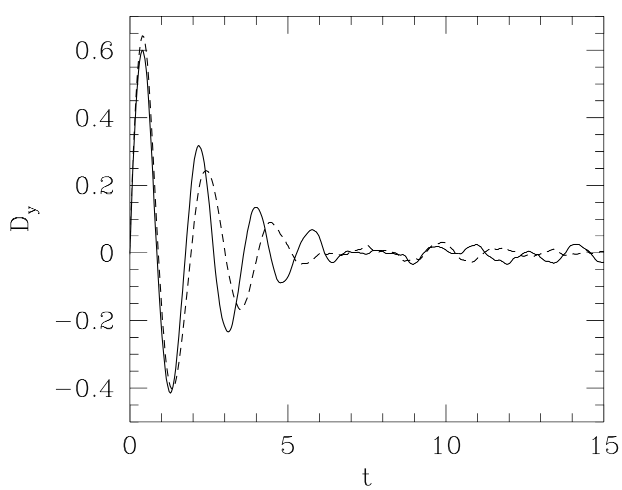

FIG. 4. Time evolution of the longitudinal dipole-moments D_y for a system of $N = 500$ nucleons at energy $E = N$. (Full line: system with two-body interaction; dashed line: mean-field model.) Time and dipole-moment are given in units of $ma^2/\hbar$ and a, respectively.